MATHEMATICS

Class VI (GANITA PRAKASH) NCERT Solutions

Chapter 1 Patterns in Mathematics

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What is Mathematics?

❖ Figure it out (Page 2)

1. Can you think of other examples where mathematics helps us in our everyday lives?

Answer: Yes, there are many examples where mathematics helps us in our everyday lives.

- ❖ Shopping
- ❖ Traveling
- ❖ Cooking
- ❖ Time Management
- ❖ Banking

2. How has mathematics helped propel humanity forward? (You might think of examples involving: carrying out scientific experiments; running our economy and democracy; building bridges, houses or other complex structures; making TVs, mobile phones, computers, bicycles, trains, cars, planes, calendars, clocks, etc.)

Answer:

Building Structures: Engineers use mathematics to ensure structural stability while building bridges or complex structures

Technology Development

Scientific and Medical Research

Running Economies: To test theories, perform research, or understand trends and predictions for elections, counting votes and economic planning.

Space Exploration

Scientific experiments: Find mathematical relationships between the things they were observing.

Economy and democracy: To test theories, perform research, or understand trends and predictions for elections, counting votes and economic planning.

Building bridges, houses, or complex structures: Engineers use mathematics to ensure structural stability while building bridges or complex structures.

❖ Patterns and Numbers

Table 1: Examples of number sequences

1, 1, 1, 1, 1, 1, 1, ...	**(All 1's)**
1, 2, 3, 4, 5, 6, 7, ...	**(Counting numbers)**
1, 3, 5, 7, 9, 11, 13, ...	**(Odd numbers)**
2, 4, 6, 8, 10, 12, 14, ...	**(Even numbers)**
1, 3, 6, 10, 15, 21, 28, ...	**(Triangular numbers)**
1, 4, 9, 16, 25, 36, 49, ...	**(Squares)**
1, 8, 27, 64, 125, 216, ...	**(Cubes)**
1, 2, 3, 5, 8, 13, 21, ...	**(Virahānka numbers)**
1, 2, 4, 8, 16, 32, 64, ...	**(Powers of 2)**
1, 3, 9, 27, 81, 243, 729, ...	**(Powers of 3)**

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❖ Page 3. Figure it Out

1. Can you recognise the pattern in each of the sequences in Table 1?

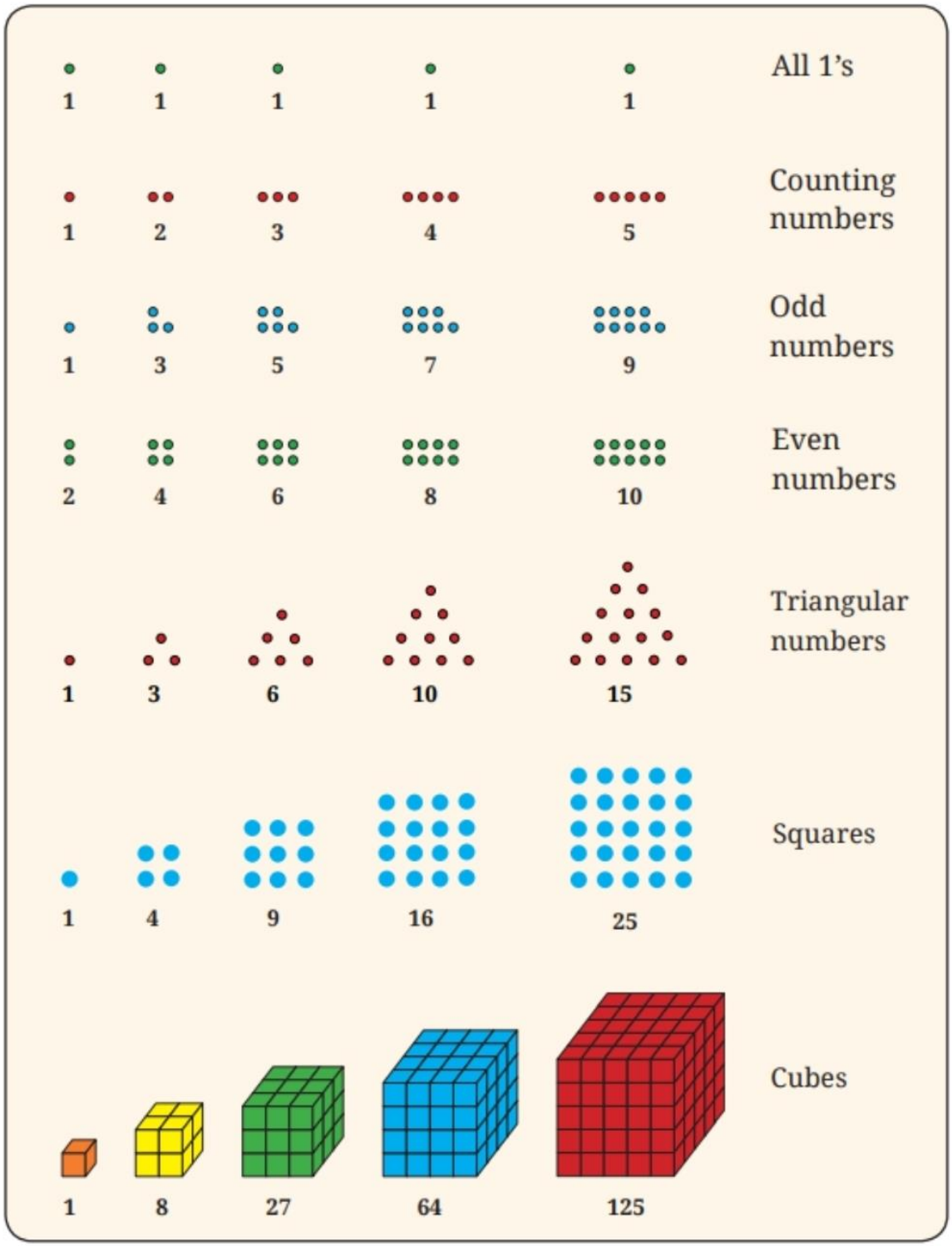
Answer: Yes

2. Rewrite each sequence of Table 1 in your notebook, along with the next three numbers in each sequence! After each sequence, write in your own words what is the rule for forming the numbers in the sequence.

Answer:

- 1, 1, 1, 1,1,**1,1,1,** ... (All 1's sequences). In the sequence, each number in the sequence is always 1.
- 1, 2, 3, 4, 5, 6, 7, **8, 9, 10,** ... (Counting numbers sequences). In the sequence, each number increases by 1.
- 1, 3, 5, 7, 9, 11, 13, **15, 17, 19,** ... (Odd numbers) In the sequence, each number increases by 2, starting from 1.
- 2, 4, 6, 8, 10, 12, 14, **16, 18, 20,** ... (Even numbers) In the sequence, each number increases by 2, starting from 2.
- 1, 3, 6, 10, 15, 21, 28, **36, 45, 55,** ... (Triangular numbers) In the sequence, the difference between consecutive numbers increases by 1 each time (+2, +3, +4…)
- 1, 4, 9, 16, 25, 36, 49, **64, 81, 100,** ... (Squares) In the sequence, each number is the result of a natural number multiplied by itself (1×1=1, 2×2=4, 3×3=9, 4×4=16,5×5=25…).
- 1, 8, 27, 64, 125, 216, **343, 512, 729,** ... (Cubes) In the sequence, each number is the result of a natural number multiplied by itself twice (1×1×1=1, 2×2×2=8, 3×3×3=9, 4×4×4=64, 5×5×5=125...).
- 1, 2, 3, 5, 8, 13, 21, **36, 57,93,** ... (Virahānka numbers) In the sequence, each term (starting from third term) is the sum of previous two terms (1+2=3, 2+3=5, 3+5=8, 8+5=13, 13+8=21, …)
- 1, 2, 4, 8, 16, 32, 64, **128, 256, 512,** ... (Powers of 2) Each number is multiplied by 2 with itself a certain number of times (1^2=1or (1×1=1), 2^2=4 or (2×2=4), 2^3=9 or (2×2×2=8), 2^4=16 or(2×2×2×2=16), 2^5=32 or (2×2×2×2×2=32),….)
- 1, 3, 9, 27, 81, 243, **729, 2187, 6561,** ... (Powers of 3) Each number is multiplied by 3 with itself a certain number of times.

Table 2: Pictorial representation of some number sequences



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❖ Page 5: Figure it Out

1. **Copy the pictorial representations of the number sequences in Table 2 in your notebook, and draw the next picture for each sequence!**

Answer: Yes

2. **Why are 1, 3, 6, 10, 15, ... called triangular numbers? Why are 1, 4, 9, 16, 25, ... called square numbers or squares? Why are 1, 8, 27, 64, 125, ... called cubes?**

Answer:

1, 3, 6, 10, 15, … are called triangular numbers because they can be represented in the form of triangle of dots.

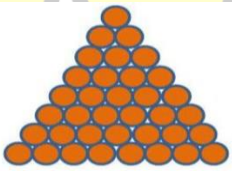
1, 4, 9, 16, 25, ... are called square numbers because these numbers can be represented in a perfect square of dots

1,8,27,64, 125, ... are called cubes because these numbers can be represented in the form of perfect cubes of blocks.

3. **You will have noticed that 36 is both a triangular number and a square number! That is, 36 dots can be arranged perfectly both in a triangle and in a square. Make pictures in your notebook illustrating this! This shows that the same number can be represented differently, and play different roles, depending on the context. Try representing some other numbers pictorially in different ways!**

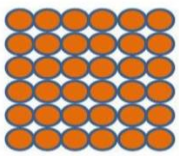
Answer:

i.



36 Dots

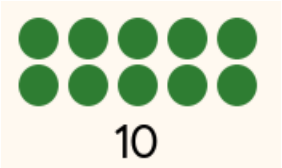
Triangular number



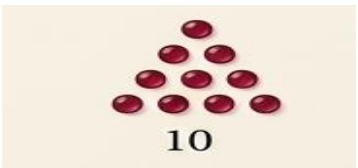
36 Dots

Square number

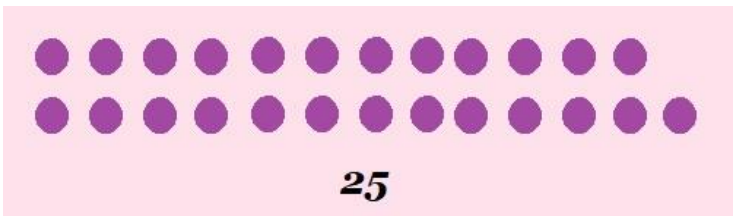
ii.



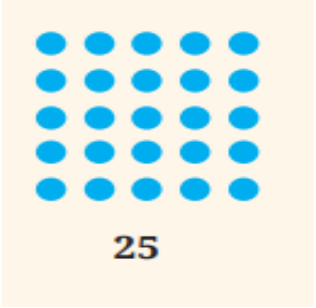
Even Number



Triangular Number

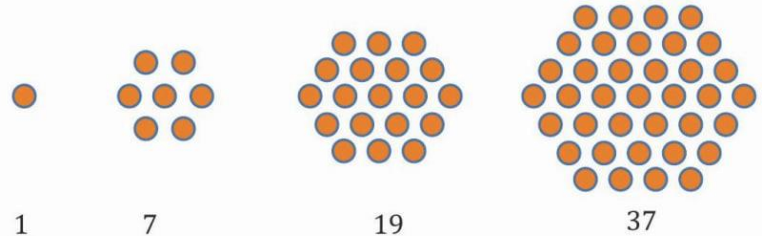


Odd Number



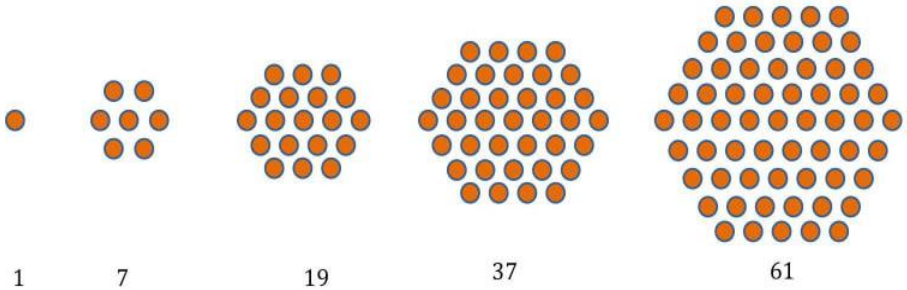
Square Number

4. What would you call the following sequence of numbers?



That's right, they are called hexagonal numbers! Draw these in your notebook. What is the next number in the sequence?

Answer:

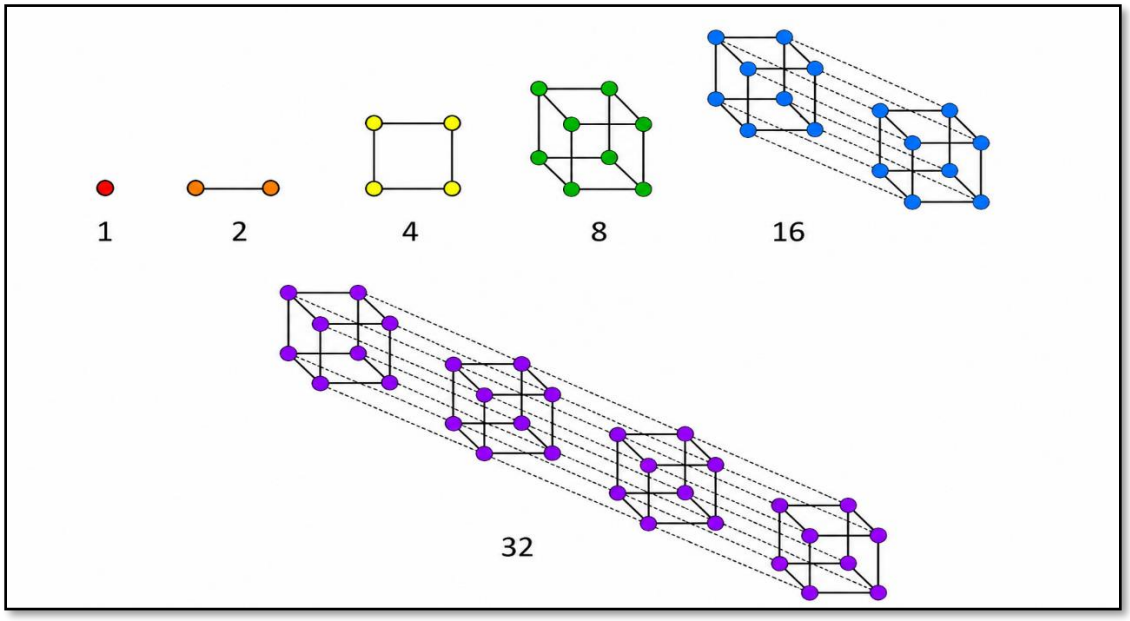


The next number in the sequence is 61

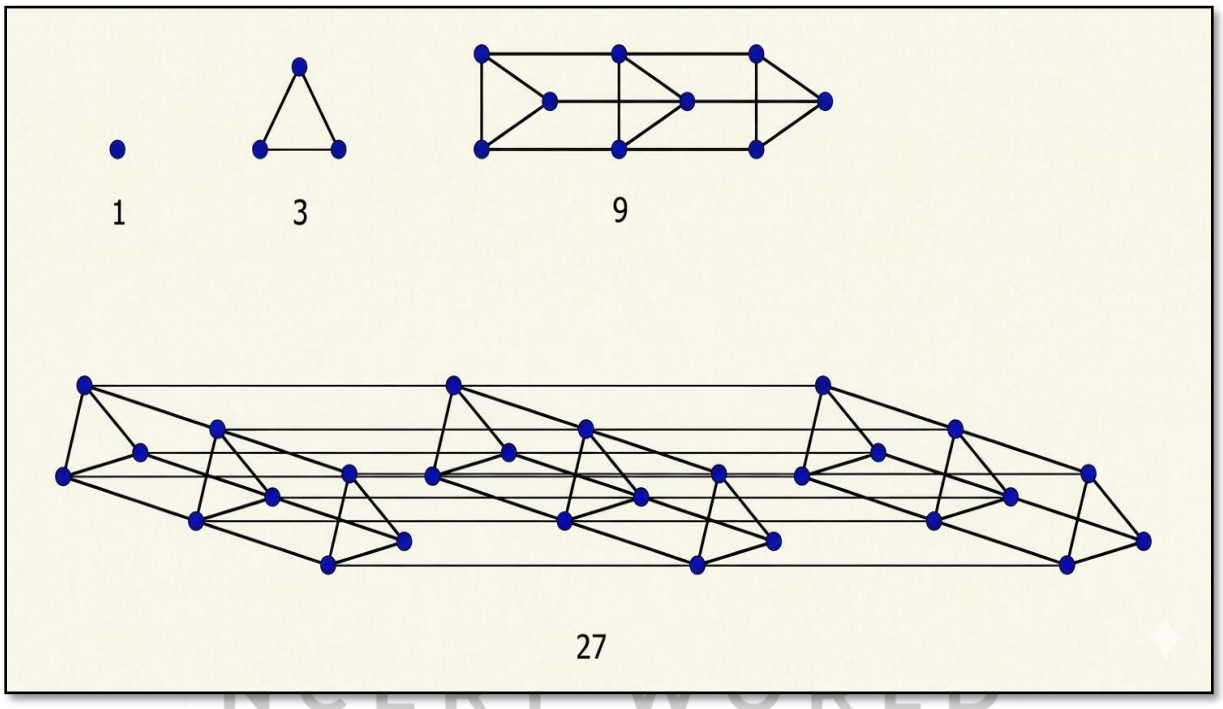
5. Can you think of pictorial ways to visualise the sequence of Powers of 2? Powers of 3?

Answer:

i. Powers of 2



ii. Powers of 3



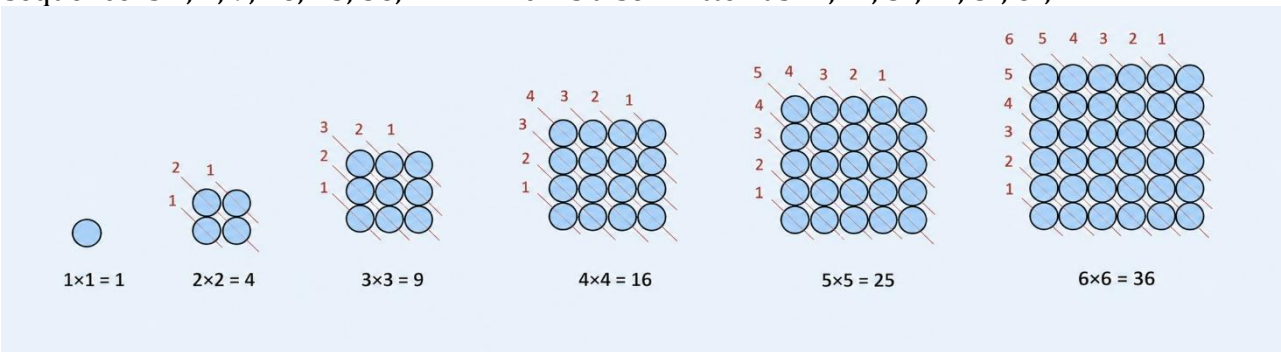
❖ Page 8-9: Figure it Out

1. Can you find a similar pictorial explanation for why adding counting numbers up and down, i.e., 1, 1 + 2 + 1, 1 + 2 + 3 + 2 + 1, ..., gives square numbers?

Answer: The counting numbers are 1, 2, 3, 4, ……

1=1
1+2+1=4
1+2+3+2+1=9
1+2+3+4+3+2+1=16
1+2+3+4+5+4+3+2+1=25
1+2+3+4+5+6+5+4+3+2+1=36

The sequence is 1, 4, 9, 16, 25, 36, ……. which is also written as 1^2, 2^2, 3^2, 4^2, 5^2, 6^2, …



2. By imagining a large version of your picture, or drawing it partially, as needed, can you see what will be the value of 1 + 2 + 3 + …… + 99 + 100 + 99 + …… + 3 + 2 + 1?

Answer:

1=1

$1+2+1=4=2^2$

$1+2+3+2+1=9=3^2$

$1+2+3+4+3+2+1=16=4^2$

So

$1+2+3+\ldots\ldots+99+100+99+\ldots\ldots+3+2+1=100^2=100\times100=10000$

3. Which sequence do you get when you start to add the All 1's sequence up? What sequence do you get when you add the All 1's sequence up and down?

Answer:

i. All 1's Sequence is 1, 1, 1, …….

Adding,

1

1+1 = 2

1+1+1=3

1+1+1+1=4

1+1+1+1+1=5, ……,

∴ The sequence is 1, 2, 3, 4, 5, ……, which are natural numbers.

ii. Adding all 1's sequence up and down

1

1+1=2

1+1+1=3

1+1+1+1=4

1+1+1+1+1=5, ……..

∴The sequence is 1, 2, 3, 4, 5, …….. which are again natural numbers.

4. Which sequence do you get when you start to add the Counting numbers up? Can you give a smaller pictorial explanation?

Answer:

Counting numbers are 1, 2, 3, 4, 5, 6, 7, …….

Adding Counting numbers up

1

1+2 = 3

1+2+3 = 6

1+2+3+4 = 10

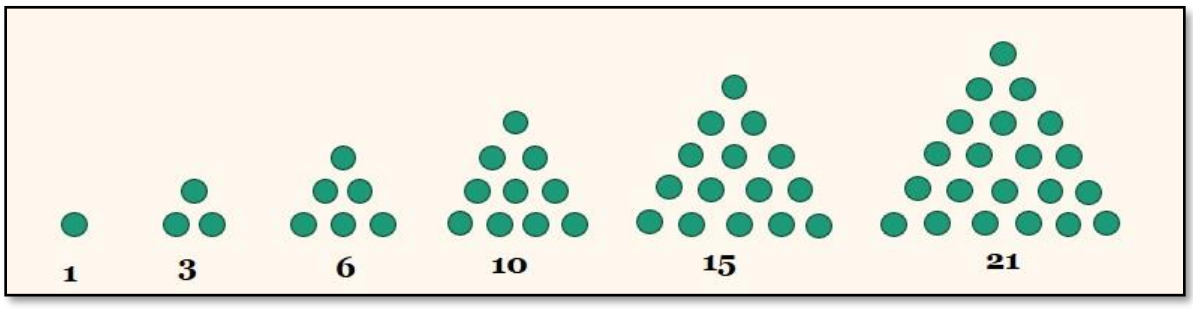
1+2+3+4+5 = 15

1+2+3+4+5+6 = 21

1+2+3+4+5+6+7 = 28

1+2+3+4+5+6+7+8= 36

The Sequence is: 1, 3, 6, 10, 15, 21, 28, 36, …….. which are triangular numbers.



5. What happens when you add up pairs of consecutive triangular numbers? That is, take 1 + 3, 3 + 6, 6 + 10, 10 + 15, ... Which sequence do you get? Why? Can you explain it with a picture?

Answer:

The triangular numbers are 1, 3, 6, 10, 15, 21, ...

1 = 1

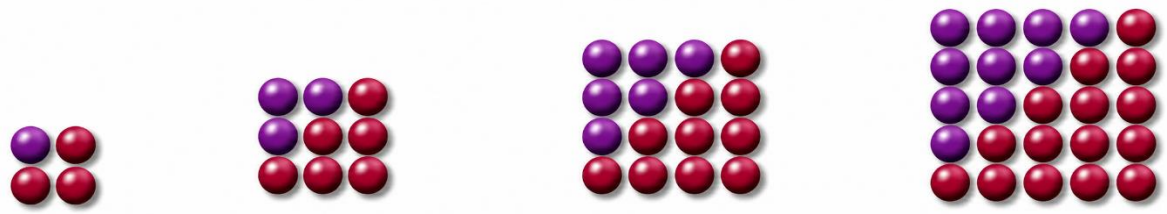
1+3 = 4

3+6 = 9

6+10 = 16

10+15 = 25

The sequence is 1, 4, 9, 16, 25, ... which are square numbers.



6. What happens when you start to add up powers of 2 starting with 1, i.e., take 1, 1 + 2, 1 + 2 + 4, 1 + 2 + 4 + 8, ...? Now add 1 to each of these numbers—what numbers do you get? Why does this happen?

Answer: Adding up powers of 2 starting with 1:

1

1 + 2 = 3

1 + 2 + 4 = 7

1 + 2 + 4 + 8 = 15

1 + 2 + 4 + 8 + 16 = 31

The sequence is 1, 3, 7, 15, 31, ...

Adding 1 to each of them,

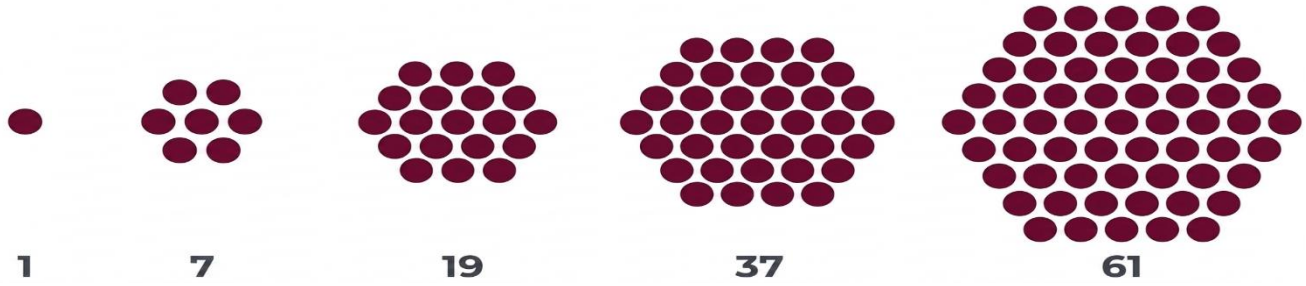
1+1 = 2

3+1= 4

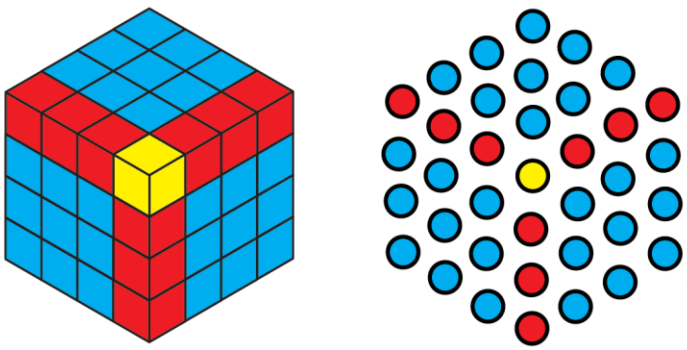
7+1 = 8
15+1 = 16
31+1 = 32, ...
The new sequence is 2, 4, 8, 16, 32, ... that can also be written as 2^1, 2^2, 2^3, 2^4, 2^5, ...
The sequence consists of powers of 2, where each term is obtained by multiplying the previous term by 2.

7. What happens when you multiply the triangular numbers by 6 and add 1? Which sequence do you get? Can you explain it with a picture?

Answer:
Triangular numbers: 1, 3, 6, 10, 15, 21, ...
Now multiply triangular numbers by 6 and adding 1, we get
1 ×6 + 1 = 7
3 × 6 + 1 = 19
6 × 6 + 1 = 37
10 × 6 + 1 = 61
15 × 6 + 1 = 91
21 × 6 + 1 = 127
28 × 6 + 1 = 163, ...
So the sequence is 7, 19, 37, 61, 91,... which are hexagonal numbers.
The pictorial representation of the abobe number is



8. What happens when you start to add up hexagonal numbers, i.e., take 1, 1 + 7, 1 + 7 + 19, 1 + 7 + 19 + 37, ...? Which sequence do you get? Can you explain it using a picture of a cube?



Answer: The hexagonal numbers are 1, 7, 19, 37, 61, 91, 169, …….

$1 = 1 = 1\times1\times1 = 1^3$

$1+7 = 8 = 2\times2\times2 = 2^3$

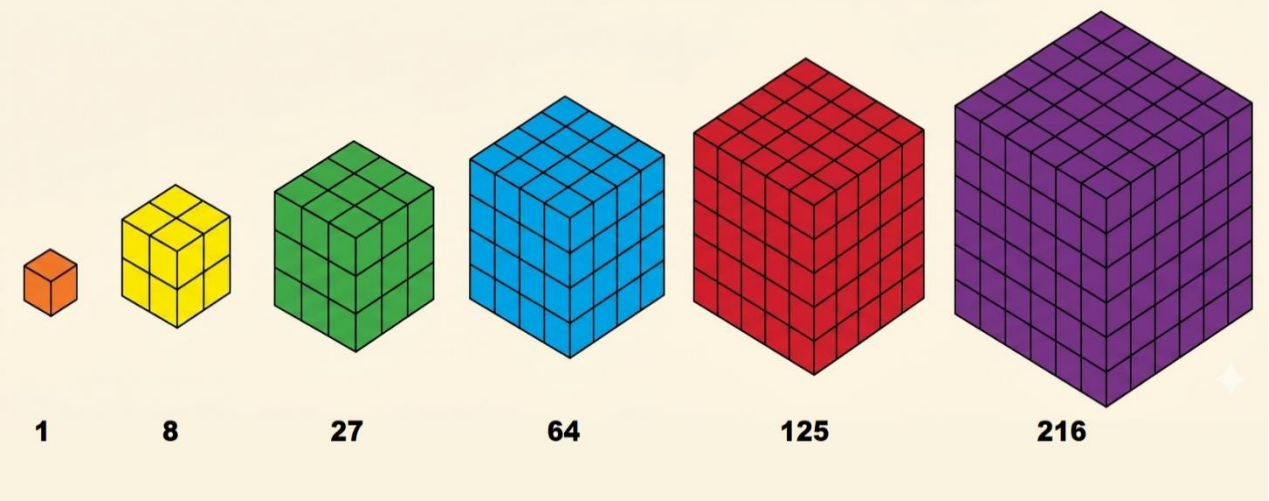
$1+7+19 = 27 = 3\times3\times3 = 3^3$

$1+7+19+37 = 64 = 4\times4\times4 = 4^3$

$1+7+19+37+61 = 125 = 5\times5\times5 = 5^3$

$1+7+19+37+61+91 = 216 = 6\times6\times6 = 6^3$……

The sequence is 1, 8, 27, 64, 125, 216, ….. which area cubes

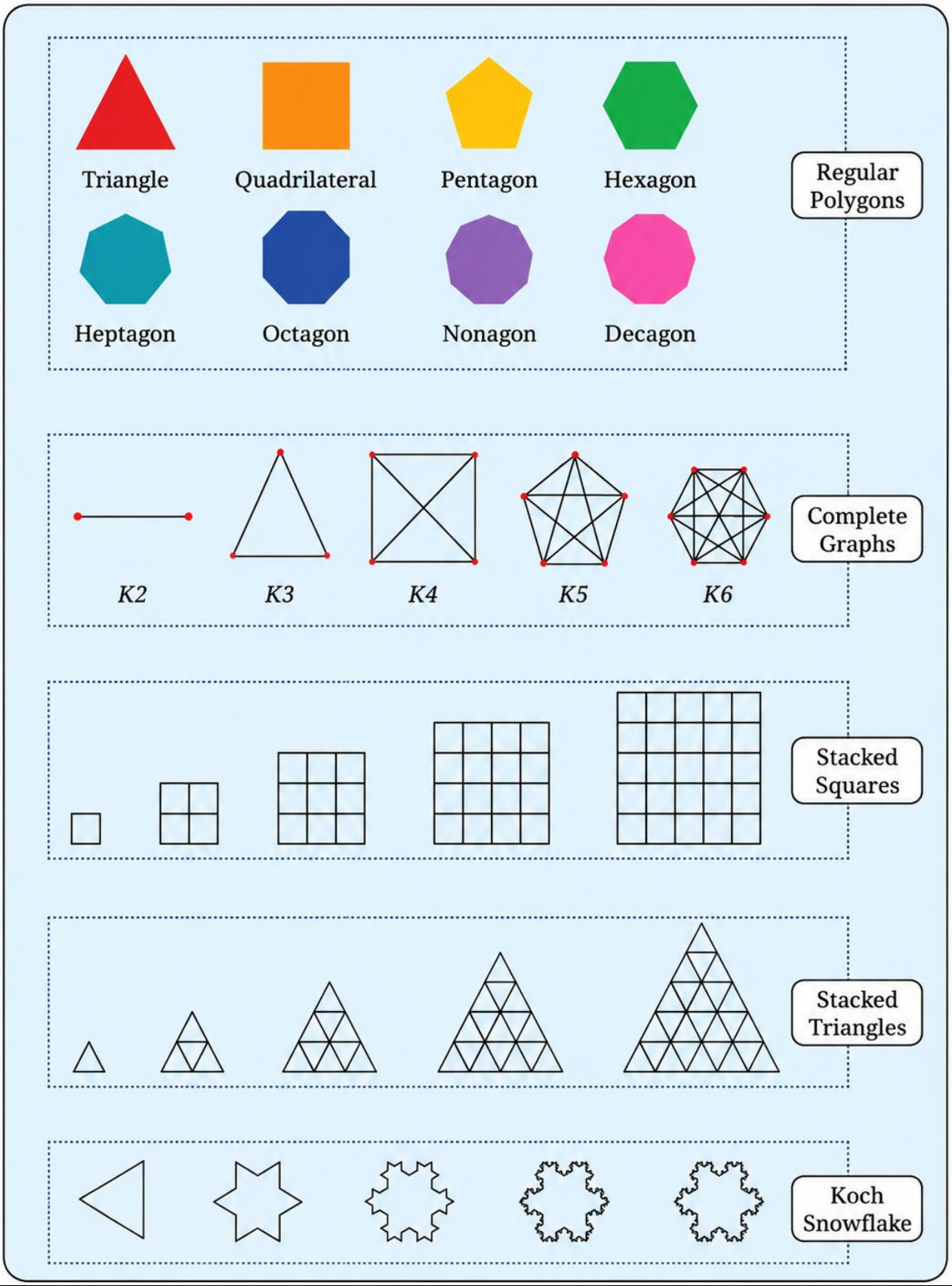


- **Patterns in Shapes**
- **Page 11**

Figure it Out

Q. Can you recognise the pattern in each of the sequences in Table 3?

Table 3 Examples of shape sequences

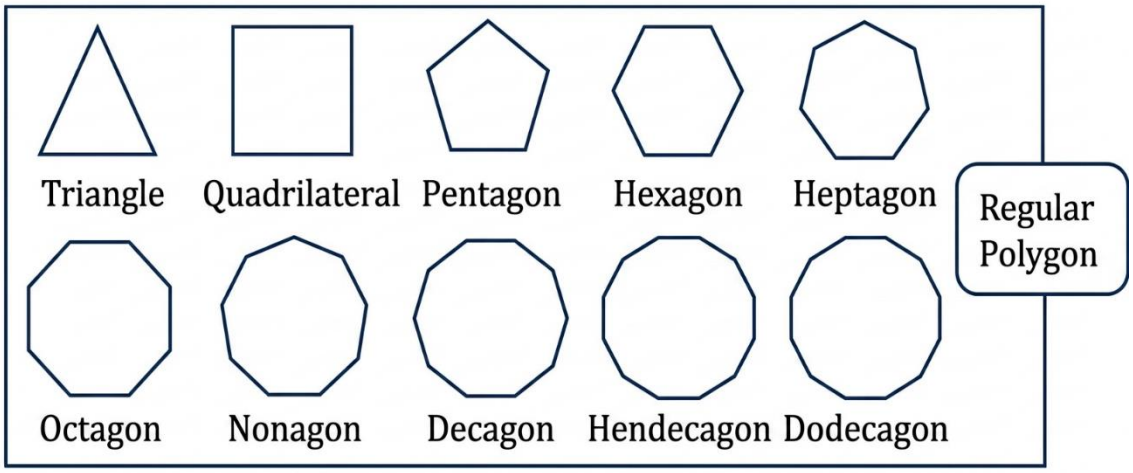


Answer: In regular Polygons Pattern: The number of sides increases by 1 each time, forming a polygon with an additional side.
Complete Graphs Pattern: The number of vertices increases by 1, and the lines connecting every vertex form a complete graph. The number of edges increases accordingly.
Stacked Squares Pattern: More squares are added as layers, increasing the total number of smaller squares.
Stacked Triangles Pattern: As new layers are added, the number of smaller triangles increases.
Koch SnowflakePattern: Each iteration adds more bumps along the edges, increasing the complexity and the number of line segments**.**

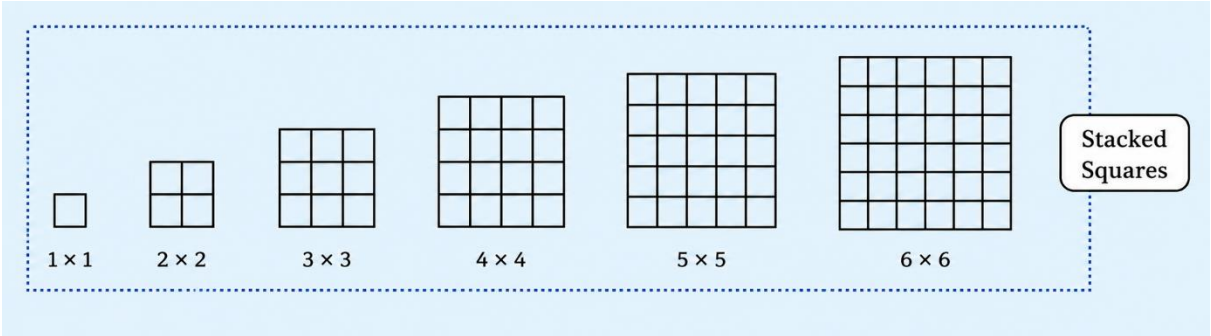
Q. Try and redraw each sequence in Table 3 in your notebook. Can you draw the next shape in each sequence? Why or why not? After each sequence, describe in your own words what is the rule or pattern for forming the shapes in the sequence.

Answer:

Regular Polygon: The next shape after the Decagon (10 sides) is the Hendecagon (11 sides). Rule: Increase the number of sides by 1



Complete graphs: After K6 (6 vertices), the next complete graph is K7. Rule: Add one more vertex and connect every vertex to all others.



Stacked Triangles: The next shape will have one more layer of triangles at the base, increasing the total number of triangles. Rule: Add another row of triangles to form a larger slacked structure.

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